

MTH 234 - Quiz 1

22 May 2015

Name: *Solutions.*

Show all your work to receive full credit on the following problems.

Consider the following vectors:

$$\vec{a} = \langle 1, 4, 1 \rangle$$

$$\vec{b} = \langle 2, 0, 0 \rangle$$

1. (5 points) Find a unit vector with the same direction as \vec{a} .

Divide by $|\vec{a}|$:

$$|\vec{a}| = \sqrt{1^2 + 4^2 + 1^2} = \sqrt{18}$$

$$\rightarrow \text{Choose } \frac{1}{\sqrt{18}} \langle 1, 4, 1 \rangle \text{ or }$$

$$\boxed{\left\langle \frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right\rangle}$$

2. (5 points) Find the cosine of the angle between \vec{a} and \vec{b} .

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \quad \text{angle between } \vec{a}, \vec{b}.$$

$$\vec{a} \cdot \vec{b} = 1 \cdot 2 + 4 \cdot 0 + 1 \cdot 0 = 2$$

$$|\vec{a}| = \sqrt{18} \quad (\text{from 1})$$

$$|\vec{b}| = 2$$

$$\gamma = \sqrt{18} \cdot 2 \cos \theta$$

$$\boxed{\cos \theta = \frac{1}{\sqrt{18}}}$$

3. (5 points) Find a vector perpendicular to both \vec{a} and \vec{b} .

Use the cross product:

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 1 \\ 2 & 0 & 0 \end{vmatrix} \\ &= \begin{vmatrix} 4 & 1 \\ 0 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} \vec{k} \\ &= 0\vec{i} - (-2)\vec{j} + (-8)\vec{k} \\ &= 0\vec{i} + 2\vec{j} - 8\vec{k}\end{aligned}$$

$\langle 0, 2, -8 \rangle$ works

$$\begin{aligned}\text{Check: } &\langle 1, 4, 1 \rangle \cdot \langle 0, 2, -8 \rangle \\ &= 8 - 8 = 0 \quad \checkmark \\ \text{and } &\langle 2, 0, 0 \rangle \cdot \langle 0, 2, -8 \rangle \\ &= 0 \quad \checkmark\end{aligned}$$

4. (5 points) Find the equation of a plane passing through the origin $(0, 0, 0)$ containing the points $(1, 4, 1)$ and $(2, 0, 0)$. (Hint: Use part (3).)

Two displacement vectors: $\vec{a} = \langle 1, 4, 1 \rangle$ and $\vec{b} = \langle 2, 0, 0 \rangle$.

\Rightarrow A normal vector to the plane is

$$\vec{n} = \vec{a} \times \vec{b} = \langle 0, 2, -8 \rangle.$$

Then use $\vec{n} \cdot \vec{r} = \vec{n} \cdot (\text{point in plane})$

with $(\text{point in plane}) = \langle 0, 0, 0 \rangle$:

$$\langle 0, 2, -8 \rangle \cdot \langle x, y, z \rangle = \langle 0, 2, -8 \rangle \cdot \langle 0, 0, 0 \rangle$$

$$2y - 8z = 0$$